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TDR-TEMPERATURE ARRAYS FOR ANALYSIS OF FIELD SOIL THERMAL PROPERTIES

by

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ABSTRACT

Harmonic analysis of diurnal soil surface and subsurface temperatures has been used to find the apparent soil thermal diffusivity in the field but has suffered from inaccuracy due to the assumption of homogeneous soil properties. Field soils usually exhibit increasing water content with depth and changing water content with time. The ability of the 3-wire TDR probe to resolve water content in a layer as thin as 0.02 m allowed creation of a system for concurrent measurement of the diurnal cycles of soil water content and temperature at seven depths in the top 0.3 m of soil. This TDR-temperature array provided data for harmonic analysis of soil thermal diffusivity and conductivity by layer. The depth variable soil water content allowed volumetric heat capacities and thermal conductivities to be calculated for a range of water contents leading to a presentation of thermal conductivity as a function of water content. This relationship compared well with published data.

INTRODUCTION

Soil thermal properties are important inputs for models of soil heat and water flux but are difficult to calculate due to the complex effects of soil particle size, shape and packing on thermal diffusivity and conductivity (*10*)[2]. Horton et al. (*11*) developed a measurement method for thermal diffusivity based on harmonic analysis. The method entailed fitting a Fourier series to the diurnal soil temperature measured at 1-h intervals at 0.01-m depth followed by the prediction of temperatures at a depth, z (0.1 m), based on the Fourier series solution to the one-dimensional heat flux problem using an assumed value of thermal diffusivity, α. The value of α was changed in an iterative fashion until the best fit between predicted and measured temperatures at z was obtained. The best fit was considered to occur when a minimum in the sum of squared differences between predicted and measured temperatures was found (i.e., minimum sum of squared error, SSE). Costello and Braud (*4*) used the same Fourier series solution and a nonlinear regression method, with diffusivity as a parameter to be fitted, for fitting the solution to temperatures measured at depths of 0.025, 0.15 and 0.3 m.

Neither (*11*) nor (*4*) addressed the dependency of diffusivity on water content or differences in water content between the different depths. Other papers have dealt with thermal diffusivity in nonuniform soils but did not result in functional relationships between thermal properties and water content, probably due to a paucity of depth-dependent soil water

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[2]Italic numbers in parentheses refer to items in the list of references.

content data (*12*, *13*). Soil water content often changes quickly with depth and horizontal distance. Moreover, diffusivity is not a single valued function of soil water content and so is difficult to directly use in modeling. The thermal conductivity, λ (J s^{-1} m^{-1}°K^{-1}), is a single-valued function of water content and is related to the diffusivity by:

$$\lambda = \alpha C_V \tag{1}$$

where the volumetric heat capacity, C_V (J m^{-3}°K^{-1}), can be calculated with reasonable accuracy from the volumetric water content, θ (m^3 m^{-3}), and the soil bulk density, ρ (Mg m^{-3}), by:

$$C_V = 2.01 \text{ x } 10^6 \rho/2.65 + 4.19 \text{ x } 10^6 \theta \tag{2}$$

for a mineral soil with negligible organic matter (*10*). Development of a conductivity vs. water content relationship using data from the harmonic method would entail twice using averaged water contents over the depth range between temperature measurements. Once to calculate C_V and again to establish the λ vs. θ relationship. The ability of time domain reflectometry (TDR) to measure water contents in layers as thin as 0.02 m (*1*, *2*) provided the basis for design of a measurement system that solves the problem of water content measurement accuracy by simultaneously providing water contents and temperatures at several depths. The objective of this study was to use data from such a measurement system to calculate soil thermal conductivity, and to determine a relationship between thermal conductivity and water content for our soil.

METHODS

The experimental site was at Bushland, TX from November 1 to December 11 (day of year 305 to 345), 1993, in the southwest lysimeter field on a Pullman silty clay loam (fine, mixed, thermic Torrertic Paleustoll) under standing wheat stubble. Thermocouples and TDR probes were installed the previous fall (1992) in 2 TDR-Temperature arrays for simultaneous measurement of soil water content and temperature. At each location probes were installed horizontally at depths of 0.02, 0.04, 0.06, 0.10, 0.15, 0.20 and 0.30 m with Cu-Co thermocouples at the same depths. Probe waveforms were automatically measured and recorded at 0.5 h intervals using an IBM PC/XT compatible computer equipped with an analog to digital conversion card. Temperatures were recorded using the same equipment. A Tektronix[3] model 1502 cable tester provided the TDR waveform output. The cable tester was modified with an optically isolated relay for electronic control of waveform output. A 16-channel multiplexer with 50-ohm characteristic impedance was designed by the author to switch the TDR signals among probes while introducing minimal signal distortion. Digital signals were provided through the computer's parallel port for both switching the multiplexer and toggling the cable tester for waveform output.

[3]The mention of trade or manufacturer names is made for information only and does not imply an endorsement, recommendation or exclusion by the USDA-Agricultural Research Service.

Three-wire TDR probes of the author's design were used. Each consisted of an epoxy resin and polymethylmethacrylate handle from which extended three parallel, type 316 stainless steel rods. The rods were spaced in a single plane at 0.03 m center to center and were 0.00318 m (nominal 1/8 inch) in diameter and 0.2 m long from the tip to the point of emergence from the handle. The outer two rods were soldered to the outer conductor of a type RG/58U coaxial cable and the inner rod was soldered to the inner conductor. The solder joints, proximal ends of the rods and distal end of the cable were encapsulated together in the handle, effectively waterproofing the connections. The three-wire configuration is semi-coaxial in nature and eliminates the need for an impedance matching transformer (balun) used with a two-wire design (*15*). In addition, the range of sensitivity above and below the plane of the rods is narrower for the 3 wire configuration than for the 2 wire configuration most commonly used in the past (*1*, *2*) allowing for better discrimination of soil water content with depth. The probes were inserted into the soil from the side of a pit so that the plane of the rods was parallel to the soil surface. The pit was backfilled with soil to approximate field bulk density. Measurements reported here began one year after probe installation.

The TDR method depends on the change in apparent dielectric constant of a soil that occurs when soil water content changes. The dielectric constant of the mineral matter in soil varies between 3 and 5. Although air may make up a large part of the soil volume, its dielectric constant is negligible by comparison with that of water which is about 80 (depending on temperature). As soil wets and dries its apparent dielectric constant, K_a, changes accordingly, though not in a linear fashion. We computed K_a as:

$$K_a = \mu^{-1}[c_o t_T/(2L)]^2 \tag{3}$$

where t_T is the two way travel time in s for the cable tester voltage pulse to travel from one impedance change to the other and back again (i.e., round trip from probe handle to end of rods), L is the distance in m between the impedance changes (i.e., TDR probe length), c_o is the speed of light, m/s, and the magnetic permittivity μ was assumed to be unity. For four fine-textured mineral soils, Topp et al. (*14*) experimentally determined a polynomial function describing the relationship between K_a and volumetric water content, θ:

$$\theta = (-530 + 292K_a - 5.5K_a^2 + 0.043K_a^3)/10^4 \tag{4}$$

The Pullman clay loam is a similar soil and Topp's equation was used. Travel times and water contents were automatically determined by the same computer program that recorded the TDR waveforms and temperatures.

The diffusion equation for heat conduction in one dimension is:

$$C\frac{\partial T}{\partial t} = \lambda \frac{\partial}{\partial z}\left[\frac{\partial T}{\partial z}\right] \tag{5}$$

where the volumetric heat capacity, C_V (J m^{-3}°K^{-1}), and the thermal conductivity, λ (J s^{-1} m^{-1}°K^{-1}), are both assumed constant in space. Vertical distance is denoted by z, time by t, and temperature by T.

The soil heat flux, G, is:

$$G = -\lambda\, \partial T/\partial z \tag{6}$$

This is Fourier's law of heat conduction for constant conductivity.

A solution to the heat conduction equation is given by a Fourier series with M terms:

$$\begin{aligned} T(z,t) = \bar{T} + \sum_{n=1}^{M} \{C_{0n}\exp[-z(0.5n\omega/\alpha)^{0.5}] \\ \sin[n\omega t + \phi_{0n} - z(0.5n\omega/\alpha)^{0.5}]\} \end{aligned} \tag{7}$$

where $\bar{T}$ is the mean temperature (the same at all depths), and the frequency ω is given in radians per unit time by $\omega = 2\pi/p$ where p is the period. The amplitude, C_{0n} (°K), and the phase angle, ϕ_{0n} (radians), will be re-defined below. For $z = 0$ (not necessarily the ground surface), Eq. 7 reduces to:

$$T(0,t) = \bar{T} + \sum_{n=1}^{M} C_{0n}\sin(n\omega t + \phi_{0n}) \tag{8}$$

which is the upper boundary condition for the solution. The lower boundary condition is:

$$T(\infty,t) = \bar{T} \tag{9}$$

Equation 8 is equivalent to:

$$T(0,t) = \bar{T} + \sum_{n=1}^{M} [A_{0n}\sin(n\omega t) + B_{0n}\cos(n\omega t)] \tag{10}$$

where A_{0n} and B_{0n} are the amplitudes (°K) of the sine and cosine terms, respectively. The phase angle and amplitude terms of Eq. 8 are related to A_{0n} and B_{0n} by $\phi_{0n} = \tan^{-1}(B_{0n}/A_{0n})$ and $C_{0n} = A_{0n}/\sin(\phi_{0n})$. A computer program was written to find the coefficients of Eq. 10 using general linear least squares regression with $M = 6$. The data used for the regression fit are referred to as the "basis" temperatures.

The fitted values of A_{0n} and B_{0n} were converted to ϕ_{0n} and C_{0n} and Eq. 7 was used to predict the temperature at a depth, z, below the depth of measurement of the basis temperatures while the value of diffusivity was changed iteratively until the sum of squared error (SSE), between predicted and actual temperatures at that depth, was minimized. The iterations were repeated several times, with progressively smaller changes between the values of diffusivity, until the value of apparent diffusivity associated with the minimum SSE was known to 4 significant digits. Temperatures at the depth z were called the "matching" temperatures.

The position for which $z = 0$ need not be taken as the soil surface. In this study z was successively set to zero for temperatures measured at depths of 0.02, 0.04, 0.06, and 0.1 m (the basis temperatures) and the respective temperatures at depths of 0.04, 0.06, 0.1, and 0.15 m were used as the matching temperatures. Data from 0.2- and 0.3-m depths were not used since the amplitude of diurnal temperature variation was close to the noise level of the data rendering the results unreliable. For measurements under bare soil, rather than wheat stubble, data from these depths would probably be useable. Water content for the layer between basis and matching temperatures was taken as the average of the mean daily water contents measured at the basis depth and the matching depth. Bulk density for a layer was linearly interpolated for the layer midpoint depth from a table of bulk density vs. depth.

Two periods with no rainfall were used, from Nov. 1-12 (DOY 305 to 316) and from Dec. 2-11 (DOY 336 to 345). On any given day the Fourier series was fit to the basis temperatures from midnight to midnight. The time passing while heat propagates from one depth to another is the phase difference, PD (days). Temperature changes beginning at midnight at the basis depth only begin to influence temperatures at the matching depth at a time PD after midnight. Therefore, estimated temperatures at the matching depth were calculated for the period from midnight plus PD on the day in question to midnight plus PD on the following day.

Furthermore, since the soil was either heating or cooling on many days, the diurnal temperatures usually did not start and stop at the same value. Since the Fourier series is constrained to return to the same value at the end of each period (1 d) there is sometimes a poor match between predicted and measured temperatures at the beginning and ending of the period. Using a 6 term series restricts the poor fit to within 2 h of the beginning and ending of the period for our data. Therefore, the SSE calculations were restricted to the period from PD + 2/24 on the given day to PD - 2/24 on the next day. Finally, since heating or cooling of the soil implies that mean diurnal temperature will vary with depth, the mean temperature in Eq. 7 was taken as the mean of measured values at the matching depth for the period PD + 2/24 to PD - 2/24 on the next day rather than the mean at the basis depth. Diffusivity values were omitted from further consideration if the SSE was higher than 0.2 $°K^2$.

RESULTS AND DISCUSSION

During the chosen periods, water content was relatively stable over time but absolute values were different for the two TDR/Temperature arrays, which were only 0.4 m apart (figures 1 and 2).

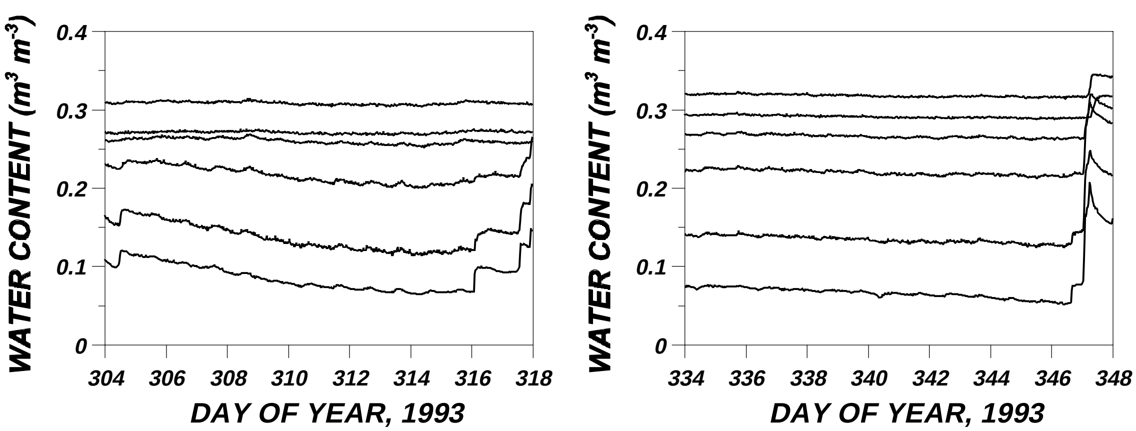

Figure 1. Water contents from TDR array 1. Note precipitation events on days 304, 316, 317, and 346. From top to bottom, lines represent 0.15, 0.2, 0.1, 0.06, 0.04, and 0.02 m depths.

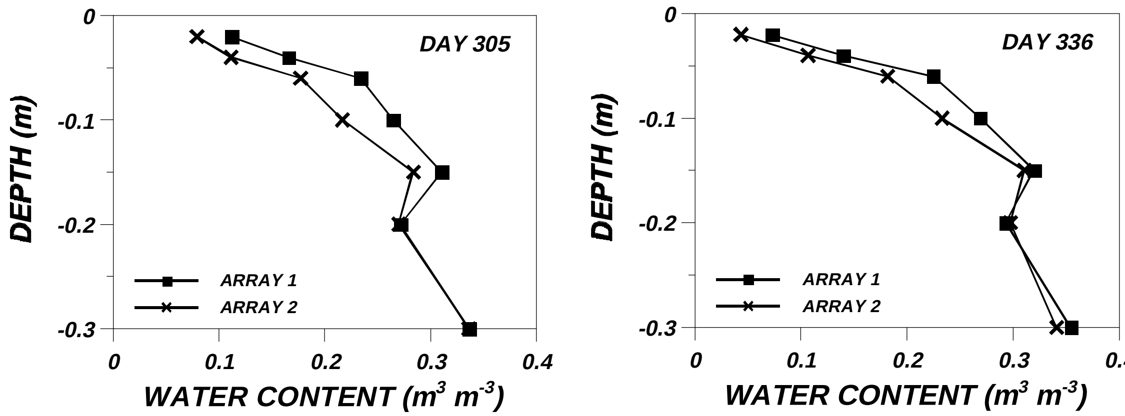

Figure 2. Profile water contents at beginning of the periods of interest.

Diurnal temperature variations were sinusoidal on most days but mean temperature increased considerably with depth since the soil was cooling during this part of the year. A typical plot of basis temperatures, matching temperatures, the fitted Fourier series and the predicted matching temperatures (using Eq. 7 and the best diffusivity value) are shown in figure 3. For this example, the phase difference was 0.052 d so the SSE was calculated starting at PD + 2/24 = 0.135 d and ended at 0.969 d as shown by the beginning and ending of the solid line through the boxes in figure 3.

About one half the data failed to pass the SSE cutoff test. The computer program correctly found the times of temperature maxima and minima at most depths most of the time. When it failed to do so, the SSE was higher than the cutoff value of 0.2 °K^2 and the diffusivity was omitted from further consideration. High SSE values also occurred when the Fourier series did not fit the basis temperatures well. This was most common at the 0.02-m depth where temperature fluctuations were most rapid.

Water content became lower and diurnal temperature variation greater at shallow depths. Due to the large diurnal variation in temperature there were very good fits and little scatter in λ values for basis and matching temperatures at 0.02- and 0.04-m depths. As the depth of basis and matching temperatures increased, the scatter in λ values increased (figure 4).

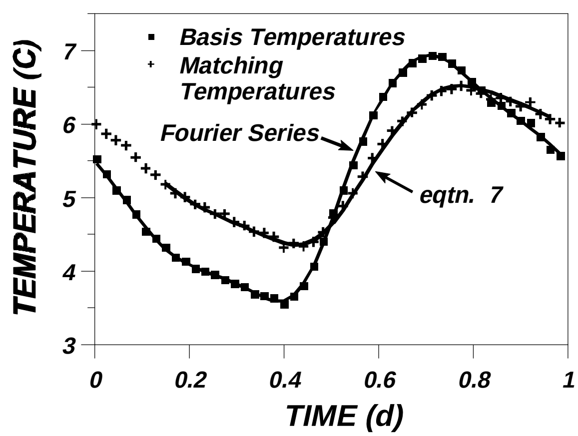

Figure 3. Fourier fit to basis temperatures at 0.06 m depth; and best predicted temperatures at 0.10 m depth using Eq. 7 compared to matching temperatures at 0.10 m depth, day 336, array 1.

The data were well fit with a linear regression:

$$\lambda = -0.07 + 3.31(\theta) \tag{11}$$

for which $r^2 = 0.84$ and standard error of estimate = 0.10 (J s^{-1} m^{-1} °K^{-1}) for 89 data pairs.

Equation 11 is not valid outside the observed range of water contents. Campbell (*3*) presented an equation that showed nonlinearity for thermal conductivity vs. water content at low water contents

$$\lambda = A + B\theta - (A - D)\exp[-(C\theta)^E] \tag{12}$$

where *A* through *E* are parameters obtained by curve fitting. Our data for clay loam plot appropriately between Campbell's predictions for silt loam and forest litter (figure 4). Also, our data appear to follow the inflection at low water contents predicted by eqtn. 12.

We have previously used the ENWATBAL model (*8*) to predict components of the energy and water balances for corn (*7*) and bare soil (*9*). These components include latent and sensible heat flux, net radiation and soil heat and water flux. In ENWATBAL, λ has been calculated following the method of De Vries (*5*) and soil heat flux calculated for our soil for different cover conditions has been uniformly higher than measured values. This is not surprising considering the large difference between the ENWATBAL calculation of λ and our current results (figure 4). We modified ENWATBAL to use eqtn. 11 and simulated energy and water balances for three seasons of wheat (*6*). The daily soil heat flux predictions were much closer to measured values than for previous simulations, with slopes for regressions of estimated G vs. measured G close to unity and intercepts close to zero.

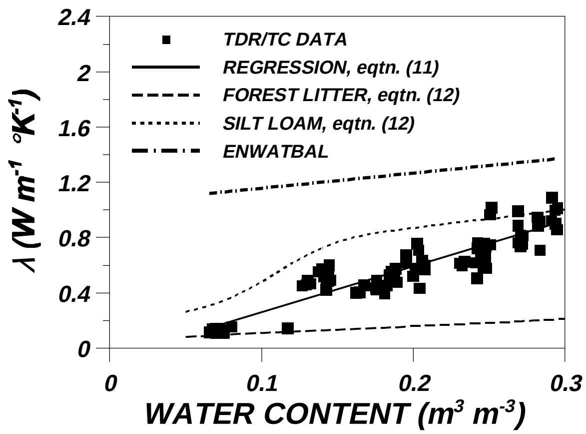

Figure 4. Comparison of data (black squares) and regression line (eqtn. 11) to predictions from eqtn. 12 for forest litter and silt loam (*3*); and, to predictions from the ENWATBAL model.

SUMMARY

In summary, we have presented a field method for determining the relationship between soil thermal conductivity and water content, a relationship heretofore usually determined in the laboratory. The method combines determination of soil water content, using TDR, and soil temperature, using thermocouples, to gather a data set that is then analyzed using statistical methods to determine the apparent thermal conductivity for a particular day and soil layer. The λ values for all layers and days are then related to the daily mean water contents of the corresponding layers and days. Results are promising in that they relate well to previous work and allow better simulation of soil heat flux using the ENWATBAL model. Although the method works well it undoubtedly can be improved. One suggestion is to use the method of Nassar and Horton (*13*), substituting water content for depth in their nonlinear regression that solves for λ vs. depth.

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