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WIND PROFILE PARAMETER ESTIMATION USING MATHCAD

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Abstract

Wind profile parameters are required to estimate aerodynamic boundary layer resistances for heat, water vapor, and momentum transfer. Traditional methods used to compute these parameters take time to program, are slow to reach solutions, and often fail to achieve solution convergence. This study evaluated the applicability of a commercial mathematics software package, MathCAD, to estimate the three parameters—friction velocity (U^*), zero-plane displacement height (d), and the roughness length, Zo—of the adiabatic log-law wind profile. The solutions were evaluated and compared to previous published solutions for several wind profiles using traditional methods. MathCAD was found to solve the wind profiles rapidly without convergence difficulty on the tested profiles. In all the comparisons, MathCAD produced almost exact duplications of previously published results. The ease, speed, and accuracy of the solution method employed in MathCAD appear to be superior to the more traditional methods.

THE WIND, air temperature, and water vapor profiles are routinely measured in micrometeorological studies of the energy balance of crops. The wind profile over a crop under adiabatic atmospheric stability is generally described (Monteith, 1973) as

$$U_z = (U^*/k) \ln[(Z - d)/Zo] \quad [1]$$

where U_z is the wind speed in meters/second at the elevation Z in meters, U^* is termed the friction velocity in meters/second, k is von Karman's constant usually taken as 0.41 (Wieringa, 1980), d is the zero-plane displacement height in meters, and Zo is the roughness length in meters. Generally, Eq. [1] will be applicable to wind speed measurements within the "inertial sub-layer" (Tennekes, 1982; Raupach and Thom, 1981). Wind speed measurements taken inside the "roughness sub-layer" result in the underestimation of the value of Zo (De Bruin and Moore, 1984). Jacobs and van Boxel (1988) defined the boundary between the roughness sublayer and the inertial sublayer, Z^*, as

$$Z^* = d + 10\ Zo. \quad [2]$$

The friction velocity is related to the shearing stress, τ, in N/meter^2 (Monteith, 1973) by

$$\tau = \rho\,(U^{*2}) \quad [3]$$

where ρ is air density in kilograms/meter^3, and the eddy momentum exchange coefficient, K_m, in meters^2/second is given by

$$K_m = k\ U^*\,(Z - d). \quad [4]$$

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Several reports have established that a linear relationship exists between d and crop height, H, in meters. These relationships, when forced through the origin, have slopes that vary from 0.61 (Uchijima and Wright, 1964) to 0.75 (Jacobs and van Boxel, 1988). Abtew et al. (1989) reported that the zero-plane displacement height, d, might be characterized as

$$d = F_c\,\bar{H} \quad [5]$$

where F_c is the fraction of the total surface area covered by roughness elements and $\bar{H}$ is the mean height of the individual roughness elements.

Several reports have indicated a linear relationship between Zo and $(H - d)$ with slopes, again when forced through the origin, between 0.25 to 0.37 (Seginer, 1974; Monteith, 1973; Shaw and Pereira, 1982; Jacobs and van Boxel, 1988). Abtew et al. (1989) reported that the relationship

$$Zo = 0.13\,(\bar{H} - d) \quad [6]$$

with d characterized from Eq. [5] fit roughness element observations from sand grain size to 20-m-tall trees from various literature. Clearly, Zo and d may not be independent, and the procedures used to estimate them may affect their apparent interrelationship.

Equation [1] contains three parameters of the adiabatic log-law wind profile—U^*, d, and Zo—that require characterization. These parameters can be determined as the roots of three nonlinear equations (Eq. [1]) which describe the wind speed at each of three measured elevations. More commonly, the wind speed profile parameters are determined so that the sum of squares between the predicted profile wind speeds and the measured profile wind speeds is minimized when the number of profile elevation measurements, N, is greater than three (Robinson, 1962; Covey, 1963; Stearns, 1970). The profile parameters are determined that minimize the sum of squares given as

$$SSE = \frac{\sum_{i=1}^{N} \{U_i - (U^*/k) \ln[(Z_i - d)/Zo]\}^2}{N - 1} \quad [7]$$

subject to the N constraints (one for each measurement elevation)

$$U_i \simeq (U^*/k) \ln[(Z_i - d)/Zo]. \quad [8]$$

The methods described by Robinson (1962) and Covey (1963) require an initial estimate for d, and a final value of d is then determined by iteration until some error criteria is achieved. The values of U^* and Zo are then obtained by substitution. Monteith (1973) suggested that trial and error values of d could be chosen and the resulting best fit plot between U_z and $\ln(Z - d)$ would determine the value of Zo as the intercept. The procedures used to determine d and Zo will likely affect the relationships observed between d and Zo. Robinson (1962) discussed the sensitive nature of the relationship between Zo and d.

The objective of this paper is to investigate the solution of wind profile data for the three parameters (U^*, d, and Zo) using a commercial mathematics computational software package. The solutions will be evaluated in terms of accuracy, speed, and ease of implementation for microcomputers. The solutions will be compared to published values using more traditional methods (Robinson, 1962; Covey, 1963; Monteith, 1973).

Materials and Methods

Wind profile data were taken from Monteith (1973) and from Covey (1963). The data from Monteith were hypothetical data for a tall and a short crop and were obtained by visual interpolation from the published graphs. Data included in Covey's report were from observations at O'Neill, NE in 1953 and six profiles reported for summer conditions at O'Neill, NE in 1956. The data used in the analyses are given in Table 1.

Table 1. Data used in analyzing wind profile parameter solutions.

Source	Description			Wind speed (m/s)					
Monteith (1973)	Tall crop					Elevation (m)			
				1.7	2.0	2.5	3.0	4.0	
				1.53	1.9	2.3	2.6	3.0	
	Short crop					Elevation (m)			
				0.25	0.50	1.0	2.0	4.0	
				1.94	2.19	2.46	2.67	3.0	
Covey (1963)	1953					Elevation (m)			
				0.4	0.8	1.6	3.2	6.4	
				4.092	5.853	6.737	7.592	8.375	
Covey (1963)	1956					Elevation (m)			
	Date	Time	0.25	0.5	1.0	2.0	4.0	8.0	16.0
	7/10	1905	2.99	3.73	4.32	4.97	5.69	6.39	7.29
	7/11	0605	3.40	4.11	4.74	5.43	6.07	6.89	7.80
	7/23	1905	3.10	3.77	4.37	—	5.68	6.35	7.31
	7/24	0605	2.89	3.36	3.80	4.20	5.05	5.32	5.84
	7/24	0705	4.26	5.01	5.52	6.09	7.08	7.63	8.43
	7/24	1905	2.24	2.59	2.91	3.39	3.98	4.27	5.03

Table 2. This is an example MathCAD program to solve a 5-level wind profile.

Unit System: This section defines the units.

m ≡ 1L
sec ≡ 1T

Constants:
$k \equiv 0.41$ von Karman's Constant

Variables:
U is wind speed in m/sec
Z is anemometer elevation in m
F is friction velocity in m/sec
d is zero-plane displacement height in m
Zo is roughness length in m

$F := \frac{\text{m}}{\text{sec}}$ This section defines the variables.

$d :=$ m
$Zo :=$ m

Input data (from page 89 in Monteith's book for the tall crop):

$$Z := \begin{bmatrix} 1.7\text{ m} \\ 2.0\text{ m} \\ 2.5\text{ m} \\ 3.0\text{ m} \\ 4.0\text{ m} \end{bmatrix} \qquad U := \begin{matrix} 1.53\,\frac{\text{m}}{\text{sec}} \\ 1.9\,\frac{\text{m}}{\text{sec}} \\ 2.3\,\frac{\text{m}}{\text{sec}} \\ 2.6\,\frac{\text{m}}{\text{sec}} \\ 3.0\,\frac{\text{m}}{\text{sec}} \end{matrix}$$

Given This section starts the solution.

$$U_0 \approx \left[\frac{F}{k}\right] \cdot \ln\left[\frac{Z_0 - (d)}{Zo}\right]$$ Anemometer elevation 1 equality constraint.

$$U_1 \approx \left[\frac{F}{k}\right] \cdot \ln\left[\frac{Z_1 - (d)}{Zo}\right]$$

$$U_2 \approx \left[\frac{F}{k}\right] \cdot \ln\left[\frac{Z_2 - (d)}{Zo}\right]$$

$$U_3 \approx \left[\frac{F}{k}\right] \cdot \ln\left[\frac{Z_3 - (d)}{Zo}\right]$$

$$U_4 \approx \left[\frac{F}{k}\right] \cdot \ln\left[\frac{Z_4 - (d)}{Zo}\right]$$

These are the solution constraints which will be solved to minimize the sum of squares between the equality constraints.

$$\begin{bmatrix} Fval \\ dval \\ Zoval \end{bmatrix} := \text{Minerr}\,(F,d,Zo)$$ This is the solution command.

Below are the solution results.

$Fval = 0.405\,\frac{\text{m}}{\text{sec}}$ $dval = 1.03$ m $Zoval = 0.142$ m

$i := 0\,..\,4$

$$PU_i := \left[\frac{Fval}{k}\right] \cdot \ln\left[\frac{Z_i - (dval)}{Zoval}\right]$$

corr $(U,PU) = 1$ slope $(U,PU) = 1$ r Value and slope.

intercept $(U,PU) = 1.345 \cdot 10^{-4}\,\frac{\text{m}}{\text{sec}}$ Intercept value.

$$SSE := \frac{\sum_i \left\{U_i - \left[\frac{Fval}{k}\right] \cdot \ln\left[\frac{Z_i - (dval)}{Zoval}\right]\right\}^2}{4}$$

$SSE = 1.969 \cdot 10^{-5}\,\frac{\text{m}^2}{\text{sec}^2}$ This is the error sum of squares.

MathCAD[1] (version 2.5) (MathSoft, Inc., Cambridge, MA 02139) was used to solve Equation [7] subject to N constraints (Eq. [8]) for each reported measurement elevation. MathCAD uses the Levenberg-Marquardt method (Levenberg, 1944; Marquardt, 1963) to estimate the solution values of the parameters to minimize the sum of squares between the measured wind speeds and the estimated wind speeds (MathCAD, 1989). Table 2 contains an example MathCAD program for the solution of a five-elevation profile such as in Monteith (1973). The MathCAD programs were executed on a DELL 310 microcomputer (Dell Computer Corp., Austin, TX 78759) operating an 80386 CPU chip at 20 Mhz and using an 80387 math coprocessor chip.

Results

The results are summarized in Table 3. For the data presented by Monteith (1973), the solution results were similar to those given by Monteith except that the roughness length, Zo, for the tall crop (H = 1.2 m) for the MathCAD solution was smaller (142 mm compared to 200 mm). The friction velocity was slightly less (0.41 m/s compared to 0.46 m/s), and the zero-plane displacement height from the MathCAD solution was slightly larger (1030 mm compared to 950 mm). The MathCAD solution parameters resulted in a better match between the measured wind speed profile and the estimated wind speed profile as indicated by a slope nearer to 1.0 (0.999 compared to 1.071), an intercept nearer to 0.0 (0.000 m/s compared to −0.166 m/s), a smaller error sum of squares (0.000 m^2/s^2 compared to 0.002 m^2/s^2). Similar results were obtained for the short crop (H = 0.05 m), but the zero-plane displacement height was smaller (0.1 mm compared to Monteith's value of 7 mm). The differences between the MathCAD solutions and the data in Monteith's book could partially be due to errors in reading the profile data from the graphs.

For the 1953 data from O'Neill, NE, reported in Robinson (1962) and Covey (1963), the MathCAD solutions were exactly the same as those reported by Robinson and Covey (Table 3). The lower intercept and error sum of squares given in Table 3 are a result of the use of additional digits in the calculations in the MathCAD program compared to the values from Covey (1963).

Covey (1963) reported solutions for 5 of 6 adiabatic wind speed profiles collected in 1956 at O'Neill, NE (Table 3). The MathCAD solution parameter values, U^*, Zo, and d, were virtually identical to those reported by Covey (1963) including the negative values for d which imply errors in either Z or U or nonadiabatic conditions. Deletion of the lower anemometer elevations due to the possibility of their being in the roughness sublayer rather than the inertial sublayer did not affect the computed negative d values (d values actually became smaller). The differences in the regression slope, intercept, correlation coefficient, and error sum of squares shown in Table 3 simply resulted from the extra precision in the MathCAD solution values compared to those reported by Covey (1963). The wind speed profile for 24 July 1956 at 1905 h, apparently, did not result in solution values using Covey's program. The MathCAD program solved that profile

[1] MathCAD is a trademark and copyrighted by MathSoft, Inc. Mention of a trade or product does not constitute a recommendation or endorsement for use by the USDA, nor does it imply registration under FIFRA as amended.

Table 3. Solution wind profile parameters and linear regression results and error sum of squares between the predicted wind speeds (dependent variable) and the measured wind speeds (independent variable).

Data source	Data description	Wind speed profile parameters			Linear regression results			
		U^*	d	Zo	r	Slope	Intercept	SSE
		m/s	mm	mm			m/s	m^2/s^2
Monteith (1973)								
	Tall crop	0.46	950	200	0.999	1.071	−0.166	0.002
MathCAD solution		0.41	1030	142	0.999	0.999	0.000	0.000
	Short crop	0.15	7	1	0.998	0.980	0.119	0.007
MathCAD solution		0.15	0.1	1.4	0.998	0.988	0.030	0.001
Covey (1963)								
	1953	0.472	95.3	4.3	0.999	0.999	0.003	0.000
MathCAD solution		0.472	95.3	4.3	0.999	0.999	0.000	0.000
	10 July 1956 1905 h	0.443	−90	20	0.999	0.998	0.030	0.006
MathCAD solution		0.443	−90.0	20.4	0.999	0.998	0.010	0.005
	11 July 1956 0605 h	0.463	−120	17	0.999	0.997	0.044	0.007
MathCAD solution		0.463	−119.9	17.5	0.999	0.998	0.012	0.006
	23 July 1956 1905 h	0.443	−124	20	0.999	1.000	0.022	0.010
MathCAD solution		0.442	−123.8	20.1	0.999	0.997	0.014	0.009
	24 July 1956 0605 h	0.308	−53	6.5	0.995	0.988	0.045	0.013
MathCAD solution		0.309	−52.7	6.5	0.995	0.991	0.039	0.013
	24 July 1956 0705 h	0.439	−96	6.1	0.998	0.997	0.037	0.012
MathCAD solution		0.438	−95.6	6.1	0.998	0.996	0.027	0.011
	24 July 1956 1905 h	–	<−240	–	–	–	–	–
MathCAD solution		0.330	−317.2	34.3	0.997	0.994	0.022	0.007

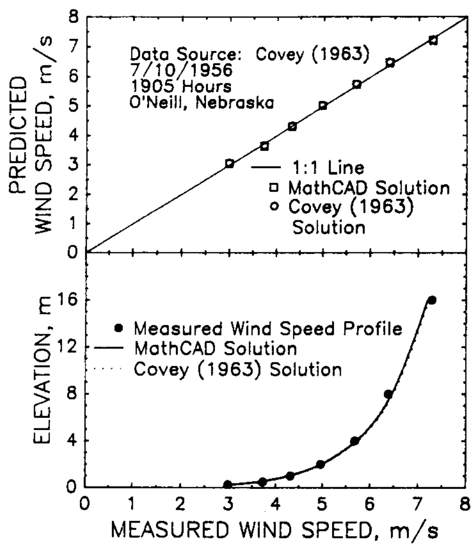

Fig. 1. Comparison of predicted and measured wind speeds for O'Neill, NE on 10 July 1956 at 1905 h using the MathCAD solution and the solution of Covey (1963).

with no apparent difficulty. Figure 1 illustrates one profile example from Covey (1963) for the 1905-h profile on 10 July 1956 showing the excellent agreement between the two solution methods.

The MathCAD programs were relatively simple to implement and develop. The programs can run from batch files operating from other MS-DOS programs and can both read data from external files and write data to external files. The solution time for the example program given in Table 2 was less than 1 s. The only problem encountered dealt with logarithm arguments when the anemometer elevation was at unit height (a height of 1 m). This problem was alleviated by defining the anemometer elevations in units of mm rather than m, which was the procedure employed in the solutions of the O'Neill, NE wind profiles.

The MathCAD solution method for solving wind profile parameters from input profile data was found to be as accurate as the methods described by Robinson (1962) and Covey (1963) which still remain the method of choice for most micrometeorology studies. The speed of the solution, rapid convergence, and ease of implementation with personal microcomputers indicates that data processing of micrometeorological wind profile data can be improved by the use of mathematical computation software packages such as MathCAD. The natural extension of the MathCAD methods to the solution on wind, temperature, and absolute humidity profiles for nonadiabatic conditions should not be difficult, although not necessarily trivial. Although many nonlinear regression methods are currently available, micrometeorologist to continue to use procedures similar to those reported by Robinson (1962) or Covey (1963). These results indicate that other methods are just as accurate and possibly simpler to implement.

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